ParCo 2003, Dresden, Germany

An Operational Semantics for Skeletons

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Outline

- Skeletons
- Semantics – motivations
- The schema of semantics
- Axioms – rules
- Example
- Concluding remarks

Skeletons

- Skeletons are language constructs
  - well-defined input-output behavior
  - parallelism exploitation patterns
  - (sometimes) can be nested
  - several prepackaged implementations
- Two main families
  - Data Parallel (*map, reduce, scan* ...)
  - Task & Stream parallel (*farm, pipeline,* ...)

Motivations

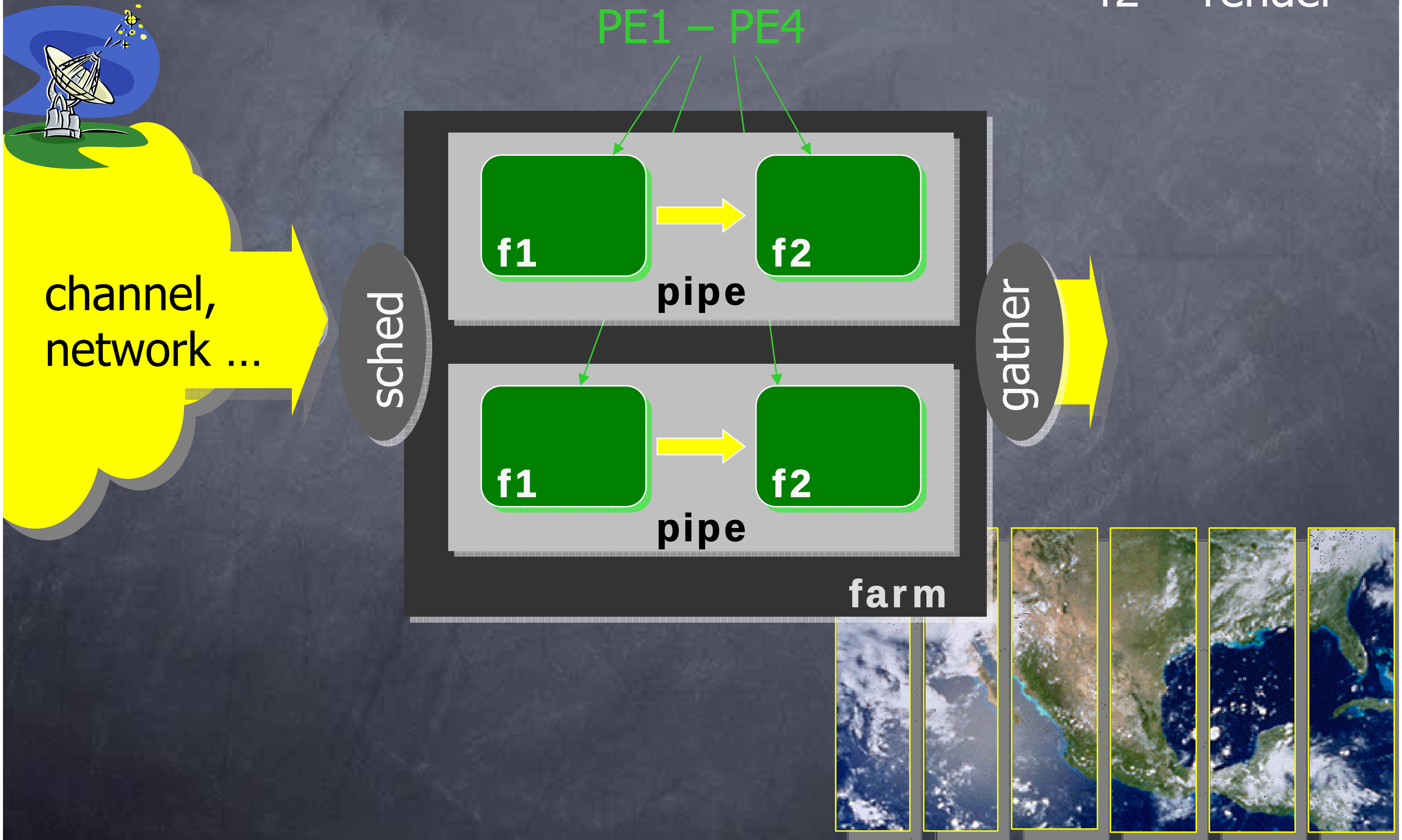
- Usually formal functional semantics, informal parallel behavior
- Describe skeletons
  - in-out relationship (functional behavior)
  - parallel behavior
  - in uniform and precise way (non steady state)
  - in structural way
- Theoretical work motivated by concrete needs
  - Enable and automate performance-driven source-to-source optimizations
  - same in/out different parallel behaviors
  - Compare different skeleton sets expressive power

e.g. with ASI

f1 = filter

f2 = render

farm (pipe (seq f1) (seq f2))



Slide 5

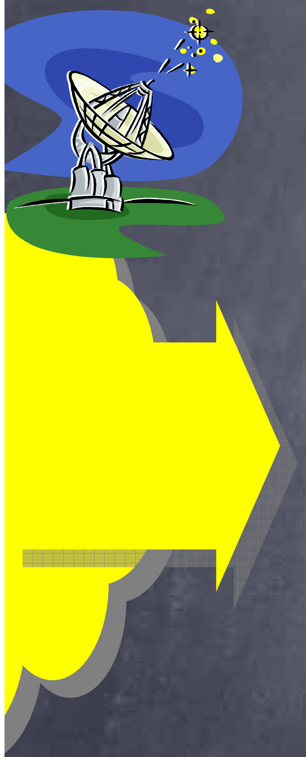
DI1

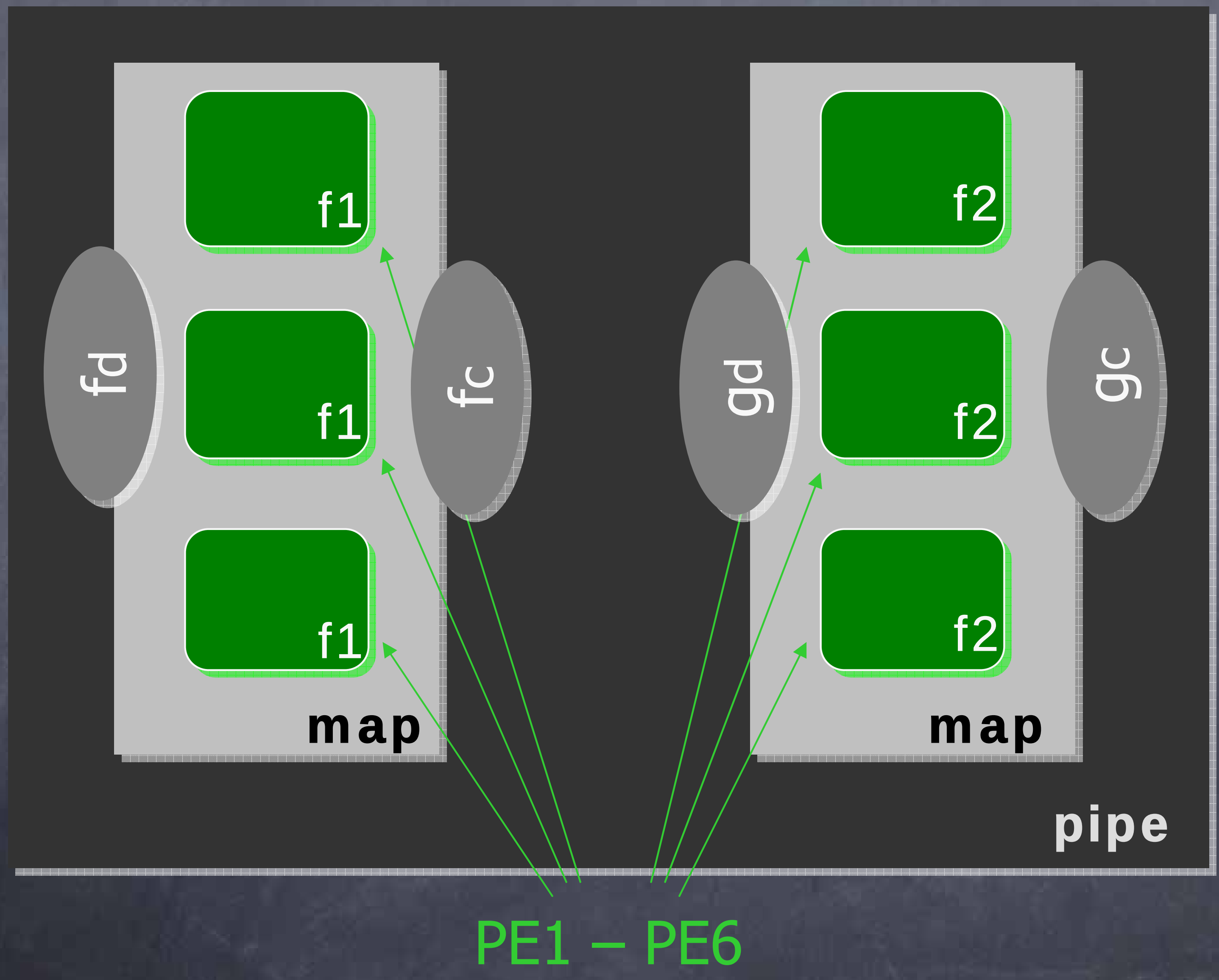
sequential source code just plugged in

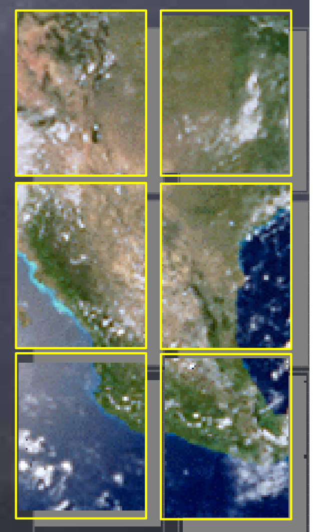
data items arrives in sequence, we cannot assume data is already distributed, data distribution cost is large, several farm scheduling policies are possible, as well as several data mappings

DipInf; 31/08/2003

pipe (map fc (seq f1) fd) (map gc (seq f2) gd)







Running example language: Lithium

- Stream and Data Parallel
  - farm, pipe
  - map, reduce, D&C, …
  - Can be freely nested
  - All skeletons have a stream as in/out
- Java-based (skeletons are Java classes)
- Implemented and running [FGCS 19(5):2003]
  http://www.di.unipi.it/~marcod/Lithium/ or sourceforge
- Macro data-flow run-time
- Support heterogeneous COWs
- Includes parallel structure optimization
  - performance-driven, source-to-source

The schema of semantics

- Axioms, three kind per skeleton:
  1. Describe skeletons within the steady state
  2. Mark the begin of stream *
  3. Manage the end of stream *

- Six rules:
  1. Two describing parallel execution (SP, DP)
     - Have a cost
  2. Four to navigate in the program structure
     - No cost, ensure strict execution order

- Look to SP/DP rules only to figure out program performance

The meaning of labels

- Label represent an enumeration of PEs
- Two kind of labels:
  - On streams represent data mapping:

    $\langle x \rangle_3$ means x is available on PE_3
  - On arrows represent computation mapping

    $\xrightarrow{4}$ means such computation is performed by PE_4
- Re-label $O_{(l,x)}$ a stream means communicate it
  - Semantics may embed an user-defined policy $O_{(l,x)}$
  - Cost depend on label (topology) and data item x (size)

Axioms (steady state)

$$\overbrace{\underline{\text{Skel}\ \text{params}}}\ \langle \overline{x}, \underline{\tau} \rangle_{\ell_1} \xrightarrow{\ell_1} \mathcal{F}\langle x \rangle_{\ell_2} \overset{3}{\overline{::}} \text{Skel params}\ \langle \tau \rangle_{\ell_3}$$

1. Apply inner skeleton $F \in$ *param* to the stream head x

2. Recur on the tail of the stream

3. Expressions 1 & 2 are joined by :: operator

a. The arrow label gets left-hand side stream label
()

b. Labels in the right-hand side may change
(stream items may be bounced elsewhere)

Lithium axioms (for stream par skeletons)

$$\text{seq}\ f\langle x,\tau\rangle_\ell \xrightarrow{\ell} \langle f\,x\rangle_\ell :: \text{seq}\ f\langle\tau\rangle_\ell$$

Embed seq code
Stream unfolded. Labels unchanged

$$\text{pipe}\ \Delta_1\Delta_2\langle x,\tau\rangle_\ell \xrightarrow{\ell} \Delta_2\mathcal{R}_{\mathcal{O}(\ell,x)}\ \Delta_1\langle x\rangle_\ell :: \text{pipe}\ \Delta_1\Delta_2\langle\tau\rangle_\ell$$

a. arrow label gets stream one – happens locally
b. label doesn't change – keep 1st stage Δ_1 locally
c. re-label *R* inserted in between 1st & 2nd stage – it will map 2nd stage elsewhere
d. tail is expected from the same source

$$\text{farm}\ \Delta\ \langle x,\tau\rangle_\ell \xrightarrow{\ell} \Delta\ \langle x\rangle_{\mathcal{O}(\ell,x)} :: \text{farm}\ \Delta\ \langle\tau\rangle_{\mathcal{O}(\ell,x)}$$

a. stream item is distributed accordingly *O* policy
b. a reference of tail of the stream follows the head

Lithium axioms (DP skeletons)

$$\mathsf{map}\ f_{\mathsf{c}}\Delta\ f_{\mathsf{d}}\langle x,\tau\rangle_\ell \xrightarrow{\ell} f_{\mathsf{c}}(\alpha\Delta)f_{\mathsf{d}}\langle x\rangle_\ell :: \ \mathsf{map}\ f_{\mathsf{c}}\Delta\ f_{\mathsf{d}}\langle\tau\rangle_\ell$$

$$\mathsf{d\&c}\ f_{\mathsf{tc}}f_{\mathsf{c}}\Delta\ f_{\mathsf{d}}\langle x,\tau\rangle_\ell \xrightarrow{\ell} \mathsf{d\&c}\ f_{\mathsf{tc}}f_{\mathsf{c}}\Delta\ f_{\mathsf{d}}\langle x\rangle_\ell :: \ \mathsf{d\&c}\ f_{\mathsf{tc}}f_{\mathsf{c}}\Delta\ f_{\mathsf{d}}\langle\tau\rangle_\ell$$

$$\mathsf{d\&c}\ f_{\mathsf{tc}}f_{\mathsf{c}}\Delta\ f_{\mathsf{d}}\langle y\rangle_\ell = \begin{cases} \Delta\ \langle y\rangle_\ell & \textit{iff}\ \ (f_{\mathsf{tc}}\ y) \\ f_{\mathsf{c}}\ (\alpha\ (\mathsf{d\&c}\ f_{\mathsf{tc}}\ f_{\mathsf{c}}\ \Delta\ f_{\mathsf{d}}))\ f_{\mathsf{d}}\ \langle y\rangle_\ell & \textit{otherwise} \end{cases}$$

Lithium rules overview

$$\frac{E_i \xrightarrow{\ell_i} E_i' \quad \forall i\ 1 \leq i \leq n \quad \wedge \quad \nexists i,j\ 1 \leq i,j \leq n,\ \ell_i = \ell_j \Rightarrow i = j}{\langle \nu \rangle_\perp :: E_1 :: \cdots E_n :: \Gamma \xrightarrow{\perp} \langle \sigma \rangle_\perp :: E_1' :: \cdots E_n' :: \Gamma}\ sp$$

$$\frac{f_{\mathsf{d}}\, \langle x \rangle_\ell \xrightarrow{\ell} \Lbag \langle y_1 \rangle_\ell, \cdots \langle y_n \rangle_\ell \Rbag \quad \Delta \langle y_i \rangle_\ell \xrightarrow{\ell} \langle z_i \rangle_\ell \quad f_{\mathsf{c}} \Lbag \langle z_1 \rangle_\ell, \cdots \langle z_n \rangle_\ell \Rbag \xrightarrow{\ell} \langle z \rangle_\ell,\ i = 1..n}{f_{\mathsf{c}}\ (\alpha\Delta)\ f_{\mathsf{d}}\, \langle x \rangle_\ell \xrightarrow{\ell} \langle z \rangle_\ell}\ dp$$

$$\frac{\Delta\ \langle x \rangle_{\ell_1} \xrightarrow{\ell_2} \langle y \rangle_{\ell_3}}{\mathcal{R}_\ell\ \Delta \langle x \rangle_{\ell_1} \xrightarrow{\ell_2} \langle y \rangle_\ell}\ relabel \qquad \frac{E_1 \xrightarrow{\ell} E_2}{\Delta\ E_1 \xrightarrow{\ell} \Delta\ E_2}\ context$$

$$\langle \sigma \rangle_{\ell_1} :: \langle \tau \rangle_{\ell_2} \xrightarrow{\perp} \langle \sigma, \tau \rangle_\perp \quad join$$

$$\langle \epsilon \rangle_\perp :: \langle \tau \rangle_{\ell_2} \xrightarrow{\perp} \langle \tau \rangle_{\ell_2} \quad join_\epsilon$$

sp rules details

$$\frac{E_i \xrightarrow{\ell_i} E'_i \quad \forall i\ 1 \le i \le n \quad \wedge \quad \exists i,j\ 1 \le i,j \le n,\ \ell_i = \ell_j \Rightarrow i = j}{\langle \nu \rangle_\perp :: E_1 :: \cdots E_n :: \Gamma \xrightarrow{\perp} \langle \sigma \rangle_\perp :: E'_1 :: \cdots E'_n :: \Gamma}\ sp$$

- Many semantics for each program
  - *i=j=1* always possible, i.e. no stream parallelism is exploited
  - All of them are *"functionally confluent"*, describe the same in-out relationship
  - All of them describe the same parallel behavior, but with different degrees of parallelism

Example (2-ways-2-stages pipeline)

farm(pipe (seq f_1) (seq f_2)) $\langle x_1, x_2, x_3, x_4, x_5, x_6, x_7 \rangle$

$\langle \epsilon \rangle_\perp$ :: farm (pipe (seq f_1) (seq f_2)) $\langle x_1, x_2, x_3, x_4, x_5, x_6, x_7 \rangle_\perp$

- Iterate the same operation on the whole stream
- pipe now disappeared
- two different labels on streams: 0 and 1
- two different labels on R : 02, 12

- Iterate the same operation on the whole stream
- farm now disappeared

pipe (seq f_1)(seq f_2)$\langle x_5 \rangle_0$ :: pipe (seq f_1)(seq f_2)$\langle x_6 \rangle_1$ :: pipe (seq f_1)(seq f_2)$\langle x_7 \rangle_0$

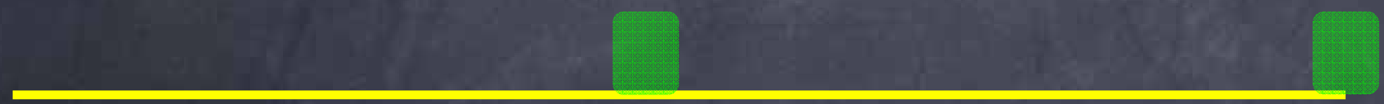
- Apply pipe inner skeletons (stages) to the item
- A re-labeling operation R is introduced in the middle

Example (continued)

$$\langle\epsilon\rangle_\perp :: \mathsf{seq}\ f_2\ \mathcal{R}_{02}\ \mathsf{seq}\ f_1\ \langle x_1\rangle_0 :: \mathsf{seq}\ f_2\ \mathcal{R}_{12}\ \mathsf{seq}\ f_1\ \langle x_2\rangle_1 :: \mathsf{seq}\ f_2\ \mathcal{R}_{02}\ \mathsf{seq}\ f_1\ \langle x_3\rangle_0 :: \mathsf{seq}\ f_2\ \mathcal{R}_{12}\ \mathsf{seq}\ f_1\ \langle x_4\rangle_1 :: \mathsf{seq}\ f_2\ \mathcal{R}_{02}\ \mathsf{seq}\ f_1\ \langle x_5\rangle_0 :: \mathsf{seq}\ f_2\ \mathcal{R}_{12}\ \mathsf{seq}\ f_1\ \langle x_6\rangle_1 :: \mathsf{seq}\ f_2\ \mathcal{R}_{02}\ \mathsf{seq}\ f_1\ \langle x_7\rangle_0$$

- This formula no longer can be reduced by axioms
- sp rule can be applied:

  "Any rightmost sequence of expressions can be reduced provided streams exploits different labels"
- In this case the longest sequence includes two expressions, i.e. the max. par degree is 2 (matching the double-pipeline startup phase)



Example (continued)

$\langle \epsilon \rangle_\perp ::$ seq $f_2 \, \langle f_1 \, x_1 \rangle_{02} ::$ seq $f_2 \, \langle f_1 \, x_2 \rangle_{12} ::$ seq $f_2 \, \mathcal{R}_{02}$ seq $f_1 \, \langle x_3 \rangle_0 ::$ seq $f_2 \, \mathcal{R}_{12}$ seq $f_1 \, \langle x_4 \rangle_1 ::$

seq $f_2 \, \mathcal{R}_{02}$ seq $f_1 \, \langle x_5 \rangle_0 ::$ seq $f_2 \, \mathcal{R}_{12}$ seq $f_1 \, \langle x_6 \rangle_1 ::$ seq $f_2 \, \mathcal{R}_{02}$ seq $f_1 \, \langle x_7 \rangle_0$

- Due to the re-labeling we have 4 adjacent expressions exploiting different labels: 02, 12, 0, 1 – i.e. a max. parallelism degree of 4
- The step can be iterated up to the end of stream
- Max parallelism degree 4 since no more than 4 different labels appear adjacently (easy to prove)

Example (continued)

By iterating SP rule we eventually get

$$\langle \epsilon \rangle_{\perp} :: \langle f_2\,(f_1\,x_1)\,\rangle_{02} :: \langle f_2\,(f_1\,x_2)\,\rangle_{12} :: \langle f_2\,(f_1\,x_3)\,\rangle_{02} :: \langle f_2\,(f_1\,x_4)\,\rangle_{12} :: \langle f_2\,(f_1\,x_5)\,\rangle_{02} :: \langle f$$

That can be joined to form the output stream

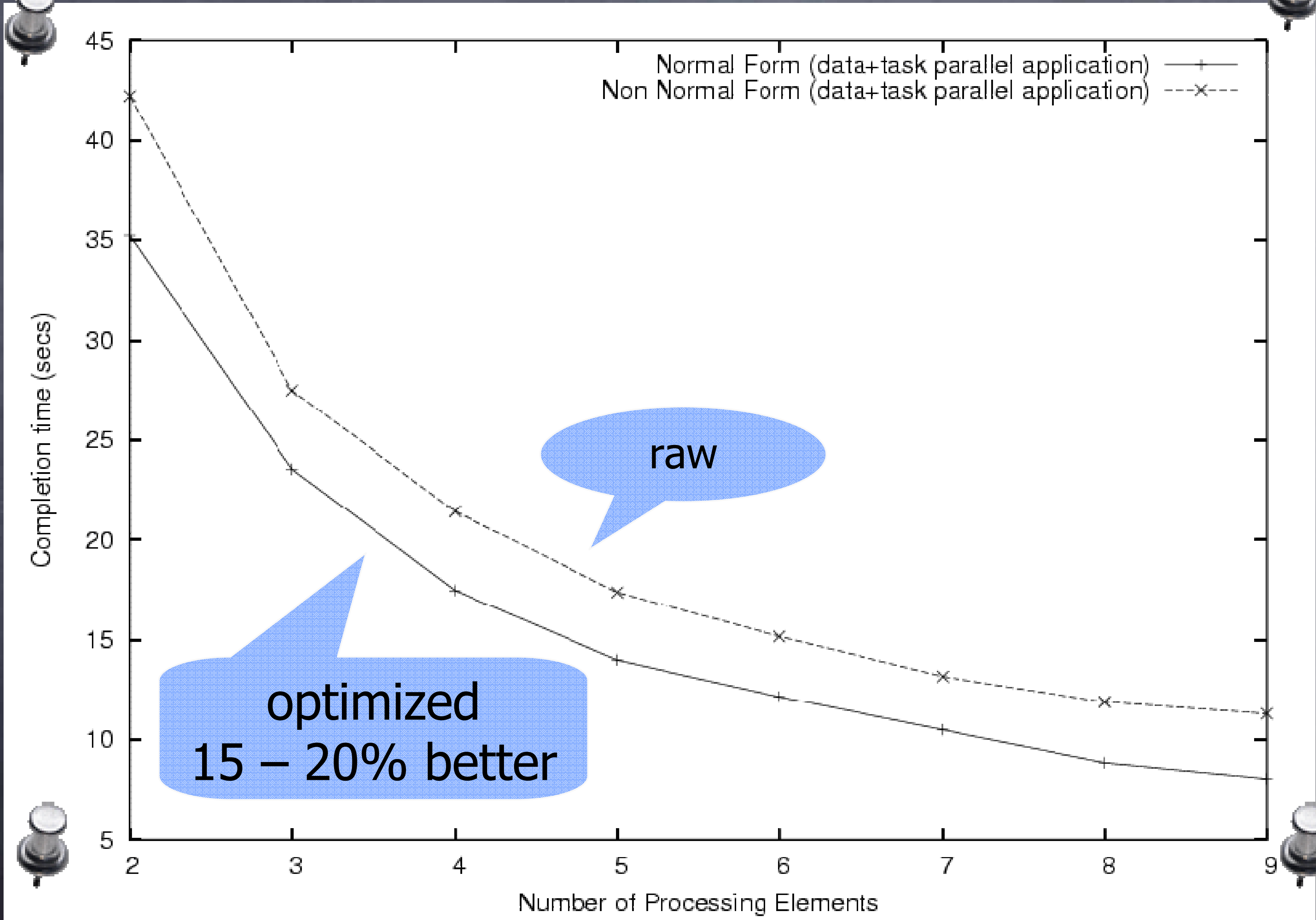
$$\langle f_2\,(f_1\,x_1),\ f_2\,(f_1\,x_2)\, f_2\,(f_1\,x_3),\ f_2\,(f_1\,x_4)\, f_2\,(f_1\,x_5),\ f_2\,(f_1\,x_6),\ f_2\,(f_1\,x_7)\,\rangle_{\perp}$$

- Count parallelism
- Count communications
- or reason about it

Summary

- Operational semantics for skeletons
  - Describes both functional and parallel behavior
  - User-defined mapping/scheduling
  - User-defined comm/comp costs
  - General, easy to extend
  - No similar results within the skeleton community
- Enable performance reasoning
  - Skeleton normal-form [PDCS99, FGCS03, web]
  - Provably correct automatic optimizations
- Formally describe your brand new skeleton and its performance

Mammography app. (lithium)



Thank you

Questions ?

www.di.unipi.it/~aldinuc

Stream skeletons

- farm
  - functionally the identity !
  - a.k.a. parameter sweeping, embarrassingly parallel, replica manager …
  - instead for some other group it is apply-to-all
- pipe
  - parallel functional composition
  - pipe $f_1\ f_2 < x >$ computes $f_2\ (\ f_1\ \ x\ )$
  - f_1 , f_2 executed in parallel on different data items

Describe skeletons

- Usually functional behavior only described
  - Parallel behavior does matter for performance
- Usually performance described by cost formulas

$$T(\text{scan } Op) = \left(\frac{n}{p} - 1\right) t_{Op} + g(p-1)\,\text{comm\_size} + l + \left(\frac{n}{p} + p - 2\right) t_{Op}$$

- Doesn't describe the behavior just the cost
- What happens if Op is parallel ?
  - Not compositional
  - handmade for each architecture
  - Data layout not described

Axioms (begin/end of the stream)

- Begin of stream marking:

$$\mathsf{Skel}\ params\ \langle x, \tau \rangle \xrightarrow{\perp} \langle \epsilon \rangle_{\perp} :: \mathsf{Skel}\ params\ \langle x, \tau \rangle_{\perp}$$

- End of stream management:

$$\mathsf{Skel}\ params\ \langle x \rangle_{\ell_1} \xrightarrow{\ell_1} \mathcal{F}\langle x \rangle_{\ell_2}$$

An example of reduction

$$
\dfrac{\dfrac{\dfrac{\mathsf{seq}\ f_1\ \langle x_1\rangle_0 \xrightarrow{0} \langle f_1\ x_1\rangle_0}{\mathcal{R}_{02}\ \mathsf{seq}\ f_1\ \langle x_1\rangle_0 \xrightarrow{0} \langle f_1\ x_1\rangle_{02}}\ \mathit{relabel}}{\mathsf{seq}\ f_2\ \mathcal{R}_{02}\ \mathsf{seq}\ f_1\ \langle x_1\rangle_0 \xrightarrow{\mathbf{0}} \mathsf{seq}\ f_2\ \langle f_1\ x\rangle_{02}}\ \mathit{context} \qquad \dfrac{\dfrac{\mathsf{seq}\ f_1\ \langle x_2\rangle_1 \xrightarrow{1} \langle f_1\ x_2\rangle_1}{\mathcal{R}_{12}\ \mathsf{seq}\ f_1\ \langle x_2\rangle_1 \xrightarrow{1} \langle f_1\ x_2\rangle_{12}}\ \mathit{relabel}}{\mathsf{seq}\ f_2\ \mathcal{R}_{12}\ \mathsf{seq}\ f_1\ \langle x_2\rangle_1 \xrightarrow{\mathbf{1}} \mathsf{seq}\ f_2\ \langle f_1\ x\rangle_{12}}\ \mathit{context}}{\begin{array}{l}\langle\epsilon\rangle_\perp :: \mathsf{seq}\ f_2\ \mathcal{R}_{02}\ \mathsf{seq}\ f_1\ \langle x_1\rangle_0 :: \mathsf{seq}\ f_2\ \mathcal{R}_{12}\ \mathsf{seq}\ f_1\ \langle x_2\rangle_1 :: \mathsf{seq}\ f_2\ \mathcal{R}_{02}\ \mathsf{seq}\ f_1\ \langle x_3\rangle_0 :: \cdots \xrightarrow{\perp} \\ \langle\epsilon\rangle_\perp :: \mathsf{seq}\ f_2 \langle f_1\ x_1\rangle_{02} :: \mathsf{seq}\ f_2\ \langle f_1\ x_2\rangle_{12} :: \mathsf{seq}\ f_2\ \mathcal{R}_{02}\ \mathsf{seq}\ f_1\ \langle x_3\rangle_0 :: \cdots\end{array}}\ \mathit{sp}
$$